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Abstract

We study the impact of technological progress on the level of employment in a vintage capital model where: i) capital and labor are gross complementary; ii) labor supply is endogenous and indivisible; iii) there is full employment, and iv) the rate of labor-saving technological progress is endogenous. We characterize the stationary distributions of vintage capital goods and the corresponding equilibrium values for employment and capital lifetime. It is shown that both variables are non-monotonic functions of technological progress indicators. Technological accelerations are found to increase employment provided innovations are not *too* radical.

Keywords: Vintage capital, Technological progress, Employment, Compensation theory

JEL numbers: C61, D21, D92, O33

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1. Introduction

Perhaps one of the most debated questions in economics is the impact of technological progress on employment (see the excellent survey by Vivarelli, 2007). In particular, labor-saving technical change has by definition a direct employment cost, which raises the question of indirect induced mechanisms which may compensate the direct job losses. Several compensation theories have been proposed. Different supply and demand side mechanisms have been put forward. Perhaps the most elementary is due to Say (1964) who argued that technological progress comes through new machines, jobs have to be created to produce these machines, which may well compensate the direct job losses. Innovations may also lead to create new goods (product innovation), giving birth to new industries and therefore to job creations (see for example Freeman and Soete, 1994). Other very known compensation theories are demand-based: for example, one would argue that as innovations decrease production costs, product prices should in turn drop provided markets are competitive, which would stimulate demand, production and therefore employment (see again Say, 1964, or more recently Smolny, 1998). One may also more straightforwardly put forward an increase in wages pushed either by productivity gains and/or by unions claim for a larger share of savings allowed by innovations (Boyer, 1998). Finally, other compensation theories target technological unemployment and the subsequent wage adjustment, which typically goes in the opposite direction to the story told just before: provided labor markets are flexible enough, wages should drop in response to unemployment, which favors job recovery (as in Addison and Teixeira, 2001).

This paper is a theoretical contribution to this literature. The key objective is to investigate the implications of vintage capital growth models concerning the covariations of employment and technological progress. The traditional vintage capital theory *à la* Solow et al. (1966) didn't pay attention to this question, employment was only treated residually as the main research question was the modernization role of investment and its impact on short-term dynamics and long-term growth. Indeed, most of the historical papers in this vein assume constant saving rate (so no preferences at all, and *a fortiori* no labor supply), the Leontief technology assumption allowing to focus on one of the two production factors (that's, capital). The recent vintage capital literature does consider full-fledged general equilibrium models with explicit preferences. Still, a vast majority of these papers ignore labor adjustment questions, and do not consider elastic labor supply. As a matter of fact, the theoretical contributions due to Benhabib and Rustichini (1991), Hritonenko and Yatsenko (1996) or Boucekkiné et al (1997,

1998) fully abstract away from employment (equilibrium or optimal) dynamics in response to embodied technical change.

Two significant exceptions are worth mentioning however. First, a few papers (like Cooley et al., 1998) include elastic labor supply and do marginally study labor adjustment to technological shocks in general equilibrium vintage capital models, the approach being fully quantitative. Second, an important literature led by Caballero and Hammour (see in particular their 1996 contribution) borrows the vintage capital structure to discuss labor market variables dynamics in frictional economics. Caballero and Hammour (1996) build on bargaining over appropriability surpluses to design inefficient labor markets within a standard vintage capital model in order to quantitatively study the pace of job creation and destruction and their cyclical properties. Boucekkine et al (1999) provide with an analytical investigation of the labor market dynamics for a special version of Caballero and Hammour's model. A few other vintage capital models designed for the quantitative analysis of inefficient labor markets, including wage inequality, have been proposed (for example, the search model proposed by Hornstein et al., 2005).

In this paper, we argue that employment covariation with technological progress is already a nontrivial question in the framework of vintage capital models with elastic labor supply, even though labor markets were efficient. To this end, we proceed in a few steps. First of all, we shed light on the nontrivial relationship between (exogenous) technological accelerations and the (endogenous) lifetime of capital goods. Second, even though one accepts the idea that capital lifetime should be shortened by innovations as new and more efficient capital goods lead to replace the oldest and obsolete ones earlier, the implications for employment (in an efficient labor market) are nontrivial. Replacement of obsolete capital amounts to closing some production units with the subsequent job losses. Whether the jobs created thanks to the new machines are enough to compensate the losses is a general equilibrium question.

At equilibrium, employment depends both on the lifetime of capital (and more broadly on the age structure of capital) and on investment in new machines (in efficiency units). Whether the latter will be stimulated enough to compensate the fall in the capital lifetime for employment to increase in response to technological accelerations is not obvious. Indeed, as wages go up in the latter case, income and substitution effects depending on preferences will drive households' consumption and labor supply responses. So once again, the general equilibrium outcomes are nontrivial. We shall investigate this issue in the indivisible labor case (Hansen, 1985), which is one of the preferred specifications in RBC theory. Though such a

specification is particular (as it implies linearity in labor), it's far from making the problem trivial as we will show.

Third, as a strong departure with respect to the vintage capital contributions cited above, we shall endogenize technological progress by introducing purposive investment in labor-saving R&D at the firm level as in Boucekkine et al (2011). As R&D expenditures are rival to investment in new capital, the general equilibrium outcomes are even less obvious. It's important to notice that the modeled R&D leads to process innovation, not to a product innovation. This is an important characteristic, especially regarding the empirical literature.

Relation to the literature

As one can infer from the motivations given above, this paper can be seen as an extension to previous work of the authors on vintage capital models (notably Boucekkine et al, 1997 and 1998, Hritonenko and Yatsenko, 1996 and 2012, or Jovanovic and Yatsenko, 2012). They can be also related to other more recent contributions of the authors like Boucekkine, Hritonenko and Yatsenko (2014a,b). Among several differences, the focus on employment and the general equilibrium appraisal of the impact of technological accelerations are distinctive enough. Compared to Caballero and Hammour (1996) and Hornstein et al. (2005), we keep labor markets efficient in our set-up (no unemployment) but add endogenous technical progress. Last but not least, our approach is mainly analytical (complemented with some numerical simulations) in contrast to the papers *à la* Cooley et al.

On the empirical ground, our findings can be discussed in the light of the large empirical literature on employment and innovations (see again the survey of Vivarelli, 2007). An interesting reference is Greenan and Guellec (2000) who worked on French manufacturing sectors over the period 1986-90. While they identified a positive correlation between innovation and employment at the firm's level with both product and process innovation, they found that such a net result does not hold at the sectoral level : compensation only works for product innovation, process innovation does generate jobs within the innovative firms but it also destroys jobs in the competing firms through a business stealing effect, leading to an overall negative effect at the sectoral level. We may interpret our vintage capital model as a model of a manufacturing sector subject to process innovation : different machines correspond to different production units running different technologies, technological accelerations lead to destroy the jobs matched with the obsolete machines at the same time as

they create jobs to operate the new machines. One of our main findings is that at stationary equilibria, process innovation does create employment provided it is not *too* radical.

2. The benchmark vintage capital growth model

As mentioned in the introduction section, we build on the seminal work of Solow et al. (1966). Optimal growth versions of the model developed in the latter have been developed by Boucekkine et al (1997), Boucekkine et al. (1998), and Hritonenko and Yatsenko (1996, 2005). We present here a decentralized equilibrium counterpart of this model, which will allow us to shed light on some original properties inherent to our decentralized framework, although the general equilibrium outcomes are identical to the optimal allocations in this model. A significant contribution of this section is to deliver a clear picture of the (mostly non-monotonic) long-term relationship between technological progress, detrended investment and the lifetime of capital when the utility function is strictly concave. Such a task has been undertaken by Boucekkine et al. (1998) using a non-standard nonlinear utility function, we shall get much clearer results in the standard case where the utility function is logarithmic. Boucekkine et al. (1997) have a linear utility function.

Firms

The major ingredient of the theory is the vintage capital technology. We shall consider the problem of a price-taking firm seeking to maximize the net profit that produces $Y(t)$ units of output, uses $L(t)$ units of labour and invests $I(t)$ into new capital:

$$\max_{I, L} \int_0^{\infty} [Y(t) - w(t)L(t) - I(t)]\mu(t)dt \quad (1)$$

$w(t)$ is the unit wage at time t , and $\mu(t)$ is the discounting factor, which depends on the stream of interest rate up to t according to:

$$\mu(t) = e^{-\int_0^t r(s)ds} \quad (2)$$

Before proceeding any further, commenting on capital costs in our competitive equilibrium set-up is worthwhile. In the profit expression given above, the price of capital is equal to I , just like one unit of output which plays the role of the numeraire. This makes sense in a one-sector model like ours. With the traditional notation inherited from Solow et al. (1966), the

price of new capital at time t is $P(t,t)$ (while $P(t,\tau)$ would designate the price at time t of a unit of capital of vintage τ), and the cost of acquiring $I(t)$ is $P(t,t) I(t)$. In competitive equilibrium, $P(t,t)$ should be equal the production cost of one unit of the new vintage t . $P(t,t)=1$ follows naturally from our one-sector setting.⁴ Accordingly, note that firms only buy the new capital goods $I(t)$ at any time t , never the older vintages. Buying older vintages would make sense in a model with learning costs as in Feichtinger et al. (2006) where it is postulated that running a new machine is harder than an old and more familiar one. We abstract away from these aspects in this paper.

The production function is a vintage capital Leontief technology: any unit of capital of vintage τ available at time t produces the same amount of output (say one unit), for any vintage τ , but operating one unit of vintage τ requires $\frac{1}{\beta(\tau)}$ units of labor. Accordingly:

$$Y(t) = \int_{a(t)}^t I(\tau) d\tau, \quad (3)$$

$$L(t) = \int_{a(t)}^t \frac{I(\tau)}{\beta(\tau)} d\tau, \quad (4)$$

where $a(t)$ is the oldest vintage in use at time t , and $L(t)$ is total labor required to operate all the machines used. In the benchmark, we consider exogenous technical progress: $\beta(t) = e^{\gamma t}$. Technological progress is Harrod neutral labor-saving at a given rate $\gamma > 0$.

The *constraints* of the firm optimization problem are given by the positivity condition

$$I(t) \geq 0, \quad (5)$$

and the standard requirement that scrapped machines cannot be reused:

$$a'(t) \geq 0, \quad a(t) \leq t. \quad (6)$$

We shall also specify the *initial conditions* as follows:

$$a(0) = a_0 < 0, \quad I(\tau) \equiv I_0(\tau), \quad \tau \in [a_0, 0]. \quad (7)$$

The optimization problem (OP) (1)-(7) includes four unknown functions I , a , Y , and L connected by equalities (3)-(4). We choose I and a as the *independent* controls of the OP and

⁴ Needless to say, the price at t of a unit of vintage τ , $P(t,\tau)$, is not equal to 1 if $\tau < t$ as this price should reflect the decreasing pattern of quasi-rents associated to the use of these vintages due to obsolescence. See Solow et al. (1966), page 99.

consider the rest of the unknown functions Y and L as the *dependent (state)* variables. The necessary conditions for **an interior** extremum are:

$$\int_t^{a^{-1}(t)} \mu(\tau) [\beta(t) - w(\tau)] d\tau = \mu(t) \beta(t), \quad (8)$$

$$w(t) = \beta(a(t)) \quad (9)$$

where $a^{-1}(t)$ is the inverse function of $a(t)$.

Equality (8) is the optimal investment rule common in vintage capital models. Dividing both sides of the equality by $\mu(t) \beta(t)$, one gets on the left hand side the marginal cost of new capital (equal to 1 in our one-sector model as argued above), and on the left hand side its expected discounted revenue over its lifetime. The flow revenue is $1 - \frac{w(\tau)}{\beta(t)}$ since each unit of capital bought at t yields one unit of output, and requires $\frac{1}{\beta(t)}$ units of labor to be operated.

Equation (9) is the optimal scrapping condition: for given wage, it allows to compute the cut-off vintage index beyond which all the machines are scrapped. It simply states that it is optimal to keep on using machines until their labor productivity ($\beta(a(t))$) equals labor cost ($w(t)$).

A quick look at equations (8)-(9) is sufficient to get that the interior extremum is **not** generally implementable from $t=0$ for given prices (that is for given wage and interest rate). This point has been already made by Boucekkine, Germain and Licandro (1997) on the optimal growth version of the model. But in our framework, this property stems from different reasons. For a given sequence of wages, the scrapping condition (9) allows to determine the optimal trajectory for the vintage index $a(t)$. This in turn determines the inverse function $a^{-1}(t)$. For given interest rate data, the left and right hand sides of the optimal investment rule (8) are therefore predetermined and need not be equal. In general so, the interior solution is not implementable from $t=0$, and a transitory corner regime will set in.⁵ In this paper, we are focusing on interior and permanent regimes, and we will show in a few paragraphs that such regimes exist in general equilibrium.

⁵ A typical example of these dynamics can be found in Boucekkine, Germain and Licandro (1997).

Consumers

The consumers' block is standard: consumers consume and save out of a total income generated by initial wealth and labor income. Financial markets are perfect, and the return to financial assets is equal to the interest rate $r(t)$. For simplification, we shall consider a constant population size and logarithmic utility throughout this paper. Boucekine et al. (1998) used instead $u(c) = c^\sigma$ with $0 < \sigma < 1$, which is not standard and does not degenerate into the benchmark logarithmic case when σ goes to zero. In the benchmark model, labor supply is inelastic, normalized to 1. So the consumers face the typical program:

$$\max_c \int_0^\infty e^{-\rho t} \ln c(t) dt, \quad \rho > 0, \quad (10)$$

with the budget constraint

$$\dot{A} = r(t)A(t) + w(t) - c(t) \quad (11)$$

where $A(t)$ is the wealth of people, $A(0)$ is given, ρ is the impatience rate. The corresponding first order conditions are as usual

$$\frac{\dot{c}}{c} = r(t) - \rho \quad \text{with} \quad \lim_{t \rightarrow \infty} \phi(t)A(t) = 0 \quad (12) ,$$

where $\phi(t)$ is the co-state variable associated with the wealth accumulation equation (11).

Decentralized equilibrium

We define the equilibrium of this economy in the following traditional way.

Definition *An equilibrium for this economy is a trajectory $(I(t), Y(t), L(t), a(t), c(t), A(t), \phi(t), r(t), \mu(t), w(t), t \geq 0)$ which solves the firms optimization problem (1)-(7) and the consumers optimization problem (10)-(11) while both the good and labor markets clear.*

The latter clearing conditions are straightforward:

$$y(t) = c(t) + I(t) \quad (13)$$

$$L(t) = 1 \quad (14)$$

It's then possible to characterize the equilibrium of the economy from a system of seven simultaneous equations:

$$\frac{\dot{c}}{c} = r(t) - \rho \quad (B1)$$

$$\mu(t) = e^{-\int_0^t r(s)ds} \quad (B2)$$

$$Y(t) = \int_{a(t)}^t \beta(\tau)m(\tau)d\tau, \quad (B3)$$

$$\int_t^{a^{-1}(t)} \mu(\tau)[\beta(t) - \beta(a(\tau))]d\tau = \mu(t)\beta(t), \quad (B4)$$

$$w(t) = \beta(a(t)) \quad (B5)$$

$$Y(t) = c(t) + \beta(t)m(t) \quad (B6)$$

$$1 = \int_{a(t)}^t m(\tau)d\tau, \quad (B7)$$

with the corresponding boundary conditions, and where for convenience we change the decision variable $I(t)$ to $m(t) = I(t)/\beta(t)$. Again, the system above only considers interior solutions for the firms' optimization problem. Nonetheless, and in contrast to the partial equilibrium firms' problem, the existence of such interior regimes cannot be immediately discarded. Indeed, prices are no longer given, and the solution to the system is far from obvious. It is easy to understand why: for given past investment profile, the labor market equilibrium condition (B7) allows to compute $a(t)$, which determines wages by (B5), and features the interest rate as a solution to an advanced integral equation from the optimal (interior) investment rule (B4). Whether this equation admits a solution is another very complicated story, which goes much beyond this section. Partial solutions are provided in Boucekkine, Germain and Licandro (1997) who solved the optimal growth version under linear utility or in Boucekkine et al. (1998) who numerically investigated the optimal short term dynamics of the same optimal growth model with nonlinear utility. Here, as mentioned above, we focus on permanent interior regimes, and we show hereafter that balanced growth paths do exist.

Balanced growth

Let us assume that $t-a(t)=T=\text{const}$, $Y(t) = \bar{y}e^{\rho t}$, and $c(t) = \bar{c}e^{\rho t}$. Then $m(t)=\bar{m}$, and the system (B1)-(B7) leads to

$$r(t) = \gamma + \rho = \text{constant}, \quad (BG1)$$

$$\mu(t) = e^{-(\gamma+\rho)t} \quad (\text{BG2})$$

$$\bar{y} = \bar{m} \frac{1 - e^{-\gamma T}}{\gamma}, \quad (\text{BG3})$$

$$\frac{1 - e^{-(\rho+\gamma)T}}{\rho + \gamma} - e^{-\gamma T} \frac{1 - e^{-\rho T}}{\rho} = 1 \quad (\text{BG4})$$

$$w(t) = e^{\gamma(t-T)} \quad (\text{BG5})$$

$$\bar{y} = \bar{c} + \bar{m} \quad (\text{BG6})$$

$$I = \bar{m} T \quad (\text{BG7})$$

In contrast to the dynamic system (B1)-(B7), the system characterizing the balanced growth paths (BGP) is straightforward. Indeed, under exogenous growth, the consumption Euler equation (BG1) immediately fixes the BGP interest rate, and equation (BG4) only depends on capital lifetime T . If this equation admits a solution, then the BGP values of all the other variables can be determined accordingly following a trivial recursive scheme. For example, (BG7) determines investment (in efficiency units), then output thanks to (BG3), and finally consumption using (BG6). The equation (BG4) is actually well-behaved.

Lemma 1 *Let $\rho < 1$. For any given $0 < \gamma < 1 - \rho$, the T -equation (BG4) has a unique positive solution $0 < T$.*

Proof. Let us denote the left-hand side of equation (BG4) as $F(T) = \frac{1 - e^{-(\rho+\gamma)T}}{\rho + \gamma} - e^{-\gamma T} \frac{1 - e^{-\rho T}}{\rho}$.

Now using the properties $F(0)=0$, $F(T) \xrightarrow{T \rightarrow \infty} \frac{1}{\rho + \gamma}$, and observing that

$$F'(T) = e^{-(\rho+\gamma)T} + \gamma e^{-\gamma T} \frac{1 - e^{-\rho T}}{\rho} - e^{-\gamma T} e^{-\rho T} = \gamma e^{-\gamma T} \frac{1 - e^{-\rho T}}{\rho} > 0,$$

one can conclude that the function $F(T)$ increases from 0 to $\frac{1}{\rho + \gamma}$. Therefore, a finite solution

T to (BG4) exists only if $\frac{1}{\rho + \gamma} > 1$, or $\rho + \gamma < 1$. The solution is unique because the function

$F(T)$ is monotonic. **Q.E.D**

It's important to note here that the conditions of Lemma 1 hold by far (notably $0 < \gamma < 1 - \rho$) under realistic parametrizations of the vintage capital model. We now move to the study of the impact of technological progress along the balanced growth paths.

Technological progress, investment and the lifetime of capital

We first characterize the impact of technological accelerations on the long-term lifetime of capital.

Proposition 1 *Let T be the solution of equation (BG4) under the conditions of Lemma 1. The value $T \rightarrow \infty$ as $\gamma \rightarrow 0$ or $\gamma \rightarrow 1 - \rho$, so T decreases in γ for small γ and increases for larger γ . The value $\gamma T \rightarrow -\ln(1 - \rho)$ as $\gamma \rightarrow 0$ and γT is larger for a larger γ . If $\rho \ll 1$ and $\gamma \ll 1$, then $T \approx \sqrt{2/\gamma}$.*

Proof. If both $\rho \ll 1$ and $\gamma \ll 1$, then applying the Taylor series up to the second order to equation (BG4), we obtain $\frac{\gamma T^2}{2} \approx 1$ or $T \approx \sqrt{\frac{2}{\gamma}}$.

In order to understand relations between γ , T , and γT for an arbitrary $\rho < 1$, let us introduce the

$$\text{auxiliary } G(T, \gamma) = \frac{1 - e^{-(\rho + \gamma)T}}{\rho + \gamma} - e^{-\gamma T} \frac{1 - e^{-\rho T}}{\rho} - 1.$$

Applying the Theorem on the Implicit Function to the equality $G(T, \gamma) = 0$, we get

$$\frac{dT}{d\gamma} = - \frac{\partial G / \partial \gamma}{\partial G / \partial T} = - \frac{T(1 - \gamma e^{-(\rho + \gamma)T} / (\rho + \gamma)) - (e^{\gamma T} - e^{-\rho T})\rho / (\rho + \gamma)^2}{\gamma(1 - e^{-\rho T})}$$

Now, calculating and estimating the derivative

$$\frac{d(\gamma T)}{d\gamma} = T + \gamma \frac{dT}{d\gamma} = \frac{\rho^2 e^{-\rho T}}{(\rho + \gamma)^2 (1 - e^{-\rho T})} (e^{-(\rho + \gamma)T} - 1 - (\rho + \gamma)) > 0$$

we obtain that the value γT monotonically increases in γ for any $0 < \gamma < 1 - \rho$. Next, presenting equation (BG4) in the form

$$\frac{1 - e^{-\rho T}}{\rho} - \frac{\gamma(1 - e^{-(\rho + \gamma)T})}{\rho(\rho + \gamma)} = 1$$

we see that the value $\gamma T \rightarrow -\ln(1 - \rho) > 0$ as $\gamma \rightarrow +0$. Correspondingly, $T \rightarrow \infty$ as $\gamma \rightarrow 0$. Finally, since the value γT remains finite as $\gamma \rightarrow +0$, the above derivative $dT/d\gamma \rightarrow -\infty$ as $\gamma \rightarrow +0$.

Therefore, the value T decreases in γ for certain small values γ , but it increases for larger γ because $T \rightarrow \infty$ again at $\gamma \rightarrow (1-\rho)$. **Q.E.D**

Proposition 1 establishes the non-monotonic nature of the relationship between the rate of technological progress and the lifetime of capital. More precisely, it proves on one hand that the BGP lifetime of capital is a decreasing function of labor-saving technical progress rate, γ , when γ is small enough, which covers most of the real-life technological accelerations. This property is obtained under much less clean conditions in Boucekkine et al. (1998) who found it to hold provided $\gamma T \leq 1$ (Proposition 2, page 368). On the other hand, and in addition to this improvement, here we show clearly that the correlation between capital lifetime and the rate of labor-saving technical progress turns out to be positive for γ large enough, a property not demonstrated in Boucekkine et al (1998). The reason is the following: while an increase in γ pushes the firms to shorten the lifetime of machines to take advantage of the better characteristics of the new ones, the discount rate of their profits plays in the opposite direction as it neatly transpires from (BG2). This second effect is dominated for γ small enough but should be predominant for large γ -values. Proposition 1 provides with a simple and clear statement of this non-monotonicity, not uncovered in the previous papers on related models. Nonetheless, from now on, we shall mostly concentrate on the realistic parameterizations of the model, that is when both ρ and γ are small enough.

Once (BG4) has a unique solution T , the whole BGP is uniquely determined as explained above. The following proposition can then be stated. It clarifies, among others, how technological progress affects detrended investment, output and consumption along the BGP.

Proposition 2 *The decentralized equilibrium (BG1)-(BG7) possesses a unique BGP as T is uniquely determined by equation (BG 4). $c(t)$ is positive, at least for small γ and ρ . For these parameterizations, when γ rises, investment level increases both while both output and consumption levels go down.*

The unique pending point in the theorem is the positivity of consumption at the BGP, which is not granted for any parameterization of the model. Yet one can straightforwardly see that consumption is indeed positive when both γ and ρ are small enough. To this end, it is enough to combine (BG3) and (BG6) under Proposition 1, and notably the approximation $T \approx \sqrt{2/\gamma}$.

The impact of technological progress on investment level (or detrended investment) at the BGP derives immediately from the clearing condition of the labor market (BG7). It implies that $\bar{m} = \frac{1}{T}$, and since T is decreasing in γ when ρ and γ are small enough, one gets that \bar{m} is increasing in γ in this parametric case. That is in this benchmark model with exogenous technical progress and inelastic labor supply, a technological acceleration shortens the lifetime of capital goods and increases the level of investment. Under Proposition 1, the model has a more precise prediction in this respect: since $T \approx \sqrt{2/\gamma}$, an acceleration in γ by one percentage point leads to an increase in the investment level by half a point. Finally observe that as one departs from the case where ρ and γ are small enough, one may get the opposite picture: the discount rate effect of technological progress on profitability of investment may dominate its scrapping time effect leading to firms using machines for a longer time and investing less (in level). We shall tackle the precise quantitative conditions and implications in the last section of this paper on an enlarged model.

Here we conclude our study of the benchmark model by looking at the effect of technological progress on detrended output and consumption when ρ and γ are small enough. The impact on output comes straightforwardly from the combination of (BG3), (BG6) and (BG7) under $T \approx \sqrt{2/\gamma}$. The decrease in the lifetime of machines is so harmful for the level of output that it is not compensated by the rise in investment. It should be noted here that (detrended) output is also a decreasing function of the rate of technological progress in the standard neoclassical growth model, say the Solow model, due to decreasing returns. The result established here is of course much less obvious because the returns are no longer decreasing and specially because there are two conflicting forces in the vintage framework (shorter capital lifetime, bigger investment level) with no obvious trade-off. The same property can be established for consumption level.

We now move to the extensions announced in the introduction section. We start by endogenizing labor supply.

3. Introducing endogenous (indivisible) labor supply

In this section, we introduce endogenous labor supply into the benchmark vintage capital studied above. Traditional vintage capital theory *à la* Solow et al. (1966) does consider inelastic labor supply. The unique departure from the benchmark is the consumption block which incorporates now disutility of work, in a linear manner consistently with the framework of indivisible labor supply put forward by Hansen (1985) and Rogerson (1988):

$$\int_0^{\infty} e^{-\rho t} [\ln c - \theta N] dt \longrightarrow \max_{C, N} \quad \theta > 0 \quad (15)$$

with the budget constraint

$$\dot{A} = r(t)A(t) + w(t)N(t) - c(t), \quad (16)$$

where $A(t)$ is the wealth of people, θ measure disutility of work (or preference for leisure), $A(0)$ is given. The decision variable $N(t)$ satisfies the following constraint:

$$0 \leq N(t) \leq 1, \quad (17)$$

This is consistent with the previous attempt to endogenize labor supply in the same general frame due to Boucekkine, Hritonenko and Yatsenko (2014a). However, these authors didn't analyze the model from the point of view of compensation theory (in the sense given in the introduction).

The first order optimality conditions corresponding to the consumer problem for an interior maximum in c, N, A are

$$1/c = e^{\rho t} \lambda \quad (18)$$

$$e^{-\rho t} \theta = \lambda w \quad \text{or} \quad \theta = w/c \quad (19)$$

$$\dot{\lambda} = \lambda r \quad (20)$$

which yields, after some trivial algebra:

$$\frac{\dot{c}}{c} = r(t) - \rho \quad \text{with} \quad \lim_{t \rightarrow \infty} \phi(t) A(t) = 0 \quad (21)$$

and

$$w = \theta c. \quad (22)$$

(22) is the new optimality equation with respect to the benchmark, it characterizes optimal (interior) labor supply by equalizing its marginal cost (θ) and marginal benefit (w/c). We now study whether the endogenization of labor supply changes the economic mechanisms (and/or

their relative sizes) disentangled in the benchmark. Since the firms' block is identical, we move directly to the decentralized equilibrium. After characterizing this equilibrium, we develop the compensation mechanisms inherent in this model.

Decentralized equilibrium and compensation mechanisms at work

With respect to the benchmark model at the decentralized equilibrium (B1)-(B7), we have now one equation more, and the resulting system writes like

$$\frac{\dot{c}}{c} = r(t) - \rho \quad (C1)$$

$$\mu(t) = e^{-\int_0^t r(s) d\tau} \quad (C2)$$

$$Y(t) = \int_{a(t)}^t \beta(\tau) m(\tau) d\tau, \quad (C3)$$

$$\int_t^{a^{-1}(t)} \mu(\tau) [\beta(t) - \beta(a(\tau))] d\tau = \mu(t) \beta(t), \quad (C4)$$

$$w(t) = \beta(a(t)) \quad (C5)$$

$$Y(t) = c(t) + \beta(t) m(t), \quad (C6)$$

$$0 < N(t) = \int_{a(t)}^t m(\tau) d\tau < 1, \quad (C7)$$

$$w(t) = \Theta c(t) \quad (C8)$$

with the initial conditions (7) and $A(0)$. Equations (C1)-(C6) are identical to (B1)-(B6). (C7) is the new labor market clearing condition and it integrates the fact that labor supply, $N(t)$, is now endogenous. (C8) is the new equation characterizing optimal labor supply. As in the benchmark case, we look for interior solutions.

Before starting the analysis of the corresponding BGPs, it is worth pointing out that the addition of endogenous labor supply strikingly complicates the involved dynamic systems. In the benchmark case, for given past investment profile, the labor market equilibrium condition (B7) allows to compute $a(t)$, which determines wages by (B5), and allows to identify the interest rate as a solution to an advanced integral equation based on the optimal (interior) investment rule (B4). This partial recursive scheme is no longer possible in this extension: the

equilibrium condition for the labor market, here (C7), has two unknowns for given past investment profile, labor supply, $N(t)$, and the vintage index, $a(t)$. The full simultaneity of the dynamic system will unfortunately remain at the BGP as it is shown hereafter.

Before getting to this point, it is important to disentangle at this stage the impact of technological progress at equilibrium in the light of the principles of the traditional compensation theory. Since equations (C1)-(C6) are identical to (B1)-(B6), one might use some of the lessons obtained from the benchmark model to comment in this respect. One is that a technological acceleration will lead to higher wages and a shorter capital lifetime (when ρ and γ are small enough). Because capital and labor are gross complementary, the latter property features the negative impact of technological progress on employment through a pure obsolescence effect: obsolete machines are scrapped (in other words $a(t)$ increases in equation C7), with the subsequent job losses. However, workers fired from obsolete production units to be closed can be reallocated to the newer units to be opened. Again because of capital/labor complementarity, this positive effect plays through the size of investment in new machines as measured by $m(t)$ in (C7).

Whether the latter effect will be strong enough to compensate the job losses caused by obsolescence is largely a general equilibrium question, and one has to look at the demand side of the economy. Indeed the increase in wages following technological acceleration raises consumption (through the optimality condition C8) but have ambiguous effects on investment and output (via equations C3 and C6), and therefore on employment (again thanks to the capital/labor complementarity).

We shall evaluate the overall impact of the latter supply and demand side compensation mechanisms along the balanced growth paths, given that an analytical exploration of this issue on the system of dynamic equations (C1)-(C8) is far intractable.

Balanced growth

Let us assume that $t-a(t)=T=\text{const}$, $Y(t)=\bar{y}e^{\gamma t}$, and $c(t)=\bar{c}e^{\gamma t}$. Then $N(t)=\bar{N}$, $m(t)=\bar{m}$, $w(t)=e^{\gamma(t-T)}$, and the system (C1)-(C8) leads, after some substitutions (to eliminate the discount term $\mu(t)$ and $w(t)$), to the following system of 6 equations

$$r(t)=\gamma+\rho=\text{const}, \quad (\text{CG1})$$

$$\bar{y} = \bar{m} \frac{1 - e^{-\gamma T}}{\gamma}, \quad (\text{CG2})$$

$$\frac{1 - e^{-(\rho+\gamma)T}}{\rho + \gamma} - e^{-\gamma T} \frac{1 - e^{-\rho T}}{\rho} = 1, \quad (\text{CG3})$$

$$\bar{y} = \bar{c} + \bar{m}, \quad (\text{CG4})$$

$$\bar{N} = \bar{m} T, \quad (\text{CG5})$$

$$\Theta \bar{c} = e^{-\gamma T} \quad (\text{CG6})$$

As announced above, the BGP system is no longer recursive: while the BGP scrapping time, T , is still the solution of the same equation as in benchmark, that is (CG3) is identical to (BG4), the rest of BGP values cannot be determined recursively from T . One has to combine simultaneously (CG2), (CG4), (CG5) and (CG6) to write employment level \bar{N} in terms of T , that is

$$\bar{N} = \frac{1}{\Theta} \frac{\gamma T e^{-\gamma T}}{1 - \gamma - e^{-\gamma T}}. \quad (\text{CG7})$$

It's then easy to characterize the BGPs.

Proposition 3 *The decentralized equilibrium (CG1)-(CG6) possesses a unique BGP:*

$$\bar{m} = \bar{N} / T \quad (\text{CG8})$$

$$Y(t) = \bar{N} \frac{1 - e^{-\gamma T}}{\gamma T} e^{\gamma t}, \quad (\text{CG9})$$

$$c(t) = \frac{e^{-\gamma T}}{\theta} e^{\gamma t}, \quad (\text{CG10})$$

where T is determined by equation (BG4), and \bar{N} is given by equation (CG7). Moreover, if and only if

$$\Theta > \frac{\gamma T e^{-\gamma T}}{1 - \gamma - e^{-\gamma T}}, \quad (\text{CG11})$$

then the optimal \bar{N} is interior. The optimal \bar{N} is corner, $\bar{N}=1$, if (CG11) fails. At $\gamma < \rho < 1$, the condition (CG11) is $\Theta > (1 - \gamma)^{-1} + o(\gamma)$.

The proof is obvious. It's useful to verify that at small values of the leisure parameter Θ , the optimal $\bar{N}=1$ is boundary, i.e., people must work maximum possible hours. For larger Θ , the

optimal $\bar{N} < 1$ is interior, i.e., people can work less (but always $\bar{N} > 0$). The last condition for interior optimal labor supply uses the fact that when $\rho \ll 1$ and $\gamma \ll 1$, we have $T \approx \sqrt{2/\gamma}$. We now move to the key economic question, namely the overall impact of technological progress on employment.

Exogenous technological progress and employment

We shall focus on the case where both ρ and γ are small enough to extract analytical results. If $\rho \ll 1$ and $\gamma \ll 1$, implying $T \approx \sqrt{2/\gamma}$, optimal \bar{N} is indeed given by the approximate formula $\bar{N} \approx \frac{1 + o(\gamma)}{\Theta(1 - \gamma)}$.

In contrast to the strongly intractable dynamic system (C1)-(C8) which delivers several possible supply and demand side interactions between technical progress and employment but no clear overall correlation, things are largely simplified along the BGPs: taking all these interactions into account, BGP restrictions yield that employment is an unambiguously increasing function of the rate of technological progress when ρ and γ are small enough. Interestingly enough note that if $\gamma \rightarrow 0$, then $T \rightarrow \infty$ (no scrapping) by Proposition 1, and $\bar{N} \rightarrow 1/\Theta$. So, if the rate of technological progress is negligible, scrapping machines is not profitable but people still work and operate existing machines. If the leisure parameter Θ is smaller than unity, then the people work maximum hours.

An important conclusion of the exercise is that just like in the typical RBC model with indivisible labor, a rise in γ does increase employment in the canonical vintage capital setting we are considering. This said, this property is much less obvious here as explained repeatedly in this section. Ambiguity has a double source in our vintage capital setting. A first ambiguity comes from the fact that technological progress affects detrended investment \bar{m} (in new capital goods) through various supply side and demand side mechanisms (see the decentralized equilibrium above) which do not *a priori* yield a non-ambiguous overall impact. Next, even though one would comfortably conjecture that detrended investment would normally go up with technological accelerations, the impact on employment is still ambiguous since $\bar{N} = \bar{m} T$: in our vintage capital set-up, capital lifetime goes unambiguously down in response to labor-saving innovations, which in turn drives employment down. Our closed form (approximate) solution for \bar{N} indicates that the latter obsolescence destructive effect is

indeed dominated by the employment creative effect through investment in new and more efficient equipment \bar{m} (under labor/capital gross complementarity), which is therefore not only positive but also sizeable enough to compensate the job losses due to obsolescence.

The latter comments call for the analysis of the dependence of the optimal investment, output and consumption levels on the rate γ . In the benchmark case, we show that a technological acceleration raises investment in level but decreases the levels of output and consumption.

The following proposition shows that these properties still hold under endogenous labor supply.

Proposition 4. *If $\gamma < \rho < 1$, then the levels of optimal output and consumption are smaller for larger values of γ while investment level does increase.*

Proof. We focus here on the impact of higher g on output level, which is the less easy part of the proof. Substituting \bar{N} and $T \approx \sqrt{2/\gamma}$ into (CG9), we obtain that

$$Y(t)e^{-\gamma t} = \frac{1}{\Theta} e^{-\gamma T} \frac{1 - e^{-\gamma T}}{1 - \gamma - e^{-\gamma T}} = \frac{1}{\Theta} \frac{e^{-x} - e^{-2x}}{1 - e^{-x} - x^2/2} = f(x),$$

where $x = \gamma T$. The differentiation of $f(x)$ gives

$$f'(x) = \frac{B}{\Theta} e^{-x} \frac{(1 - e^{-x})(e^{-x} - 1 + x + x^2/2) - e^{-x}x^2/2}{(1 - e^{-x} - x^2/2)^2}$$

Hence, at small γ , $\gamma T \approx \sqrt{2\gamma}$ is also small. Presenting the exponent e^{-x} as the Taylor series, we obtain

$$f'(x) \approx -\frac{B}{2\Theta} e^{-x} \frac{x^2(1 - 3x + 4x^2/3)}{(1 - e^{-x} - x^2/2)^2}$$

Hence, $f'(x) < 0$ at small x and γ . **Q.E.D**

That's to say that the endogenous scrapping mechanism inherent to the vintage capital model is quantitatively strong enough to dominate the impact on output level of a simultaneous rise of employment and investment induced by exogenous technological accelerations. The next section evaluates whether the same conclusion holds if technological progress is endogenous (and of course costly).

4. Introducing endogenous labor-saving technological progress

In this section, we assume that firms choose the optimal lifetime of their vintage capital and invest in new capital and in adoptive and/or innovative R&D. We shut up at the moment the endogenous labor supply channel to isolate the mechanisms generated by endogenous technical change. Therefore the consumption block is identical to the one of the benchmark model. Only the firms' block is re-designed. Mathematically, this section draws on previous work by Boucekkine, Hritonenko and Yatsenko (2009, 2011, 2014b), applied to energy-saving technical progress under pollution quota. The 2009 paper explores the properties of a Solow version of the model (constant saving rate), the 2011 article solves a partial equilibrium (firm) model while the most recent one examines the optimal growth counterpart. In this section with inelastic labor supply, we study the decentralized equilibrium version of this class of models where energy is replaced by labor and the pollution quota by the labor market clearing condition. Several results of this section can be easily adapted from Boucekkine, Hritonenko and Yatsenko (2011, 2014b) but not the general equilibrium outcomes which are new.

A key modelling aspect is R&D investment decision. As outlined in the introduction, innovation may consist in adding a new product or moving to a new production process. Consistently with the traditional vintage capital setting, innovation is labor-saving, and though it is exclusively conveyed by new capital goods, we interpret it as process innovation: this is a one-sector model and the successive capital goods represent the available frontier technologies. As explained in the introduction, the related empirical literature (like Greenan and Guellec, 2000) tend to deliver that process innovations are associated with a negative effect on employment at the sectoral level. Though these innovations have a strong negative effect in our model through shutting down the obsolete production units, we show hereafter (notably in Section 5) that the overall effect on employment is positive unless the innovation is *too* radical. Last but not least, for a matter of simplification, we assume that R&D is undertaken by the same representative firm producing the final good. This is consistent with the observation that the largest part of R&D (private) expenditures are undertaken by the major companies in the automotive, ICT and pharmaceutical sectors.

The new firm problem

We shall consider the problem of a firm seeking to maximize the net profit that produces $Y(t)$ units of output, uses $L(t)$ units of labor, invests $R(t)$ into innovative and/or adoptive R&D, and invests $I(t)$ into new capital:

$$\max_{R,I,L} \int_0^{\infty} [Y(t) - w(t)L(t) - R(t) - I(t)]\mu(t)dt$$

under the same constraints (2) to (4) as the benchmark firm. The novelty is the control $R(t)$, which is the amount (in terms of final good) the firm spent in improving its labor use through adoptive or innovative R&D, and reorganization. The associated production function is

$$\frac{\beta'(\tau)}{\beta(\tau)} = \frac{bR^n(\tau)}{\beta^d(\tau)}, \quad 0 < n < 1, \quad d > 0, \quad b > 0, \quad (23)$$

Following Boucekkine, Hritonenko, Yatsenko (2011 and 2014b), we postulate that the level $\beta(\tau)$ of the labor-saving technological progress evolves endogenously according to (23), where n is the parameter of “R&D efficiency” and d is the parameter of “R&D complexity”. The constraints of the problem are identical to those of the benchmark firm problem. Initial conditions include those of the benchmark, plus a data on past $R(t)$ values because the delayed integral labor demand equation (4) requires such a data:

$$a(0) = a_0 < 0, \quad \beta(a_0) = \beta_0, \quad I(\tau) \equiv I_0(\tau), \quad R(\tau) \equiv R_0(\tau), \quad \tau \in [a_0, 0].$$

The nonlinear ODE (23) has an exact solution of the form:

$$\beta(\tau) = \left(d \int_0^{\tau} b R^n(v) dv + B^d \right)^{1/d}, \quad \tau \in [0, \infty), \quad (24)$$

where $B = \beta(0) = \left(d \int_{a_0}^0 b R_0^n(v) dv + \beta_0^d \right)^{1/d}$ using the initial conditions

The optimization problem (OP) includes six unknown functions R , β , I , a , Y , and L connected by equalities (3), (4) and (23). Following Hritonenko and Yatsenko (1996) and Yatsenko (2004), we choose R , I , and a as the *independent* controls of the OP and consider the rest of the unknown functions β , y , and L as the *dependent (state)* variables. Using again the variable change $m(t) = I(t)/\beta(t)$, one can find the following *necessary first order conditions* :

Lemma 2. *Let (R, m, a, β, Y, L) be an interior solution of the OP. Then*

$$bnR^{n-1}(t) \int_t^\infty \beta^{1-d}(\tau) m(\tau) \left[\int_\tau^{a^{-1}(\tau)} \mu(z) dz - \mu(\tau) \right] d\tau = \mu(t), \quad (25)$$

$$\int_t^{a^{-1}(t)} \mu(\tau) [\beta(t) - w(\tau)] d\tau = \mu(t) \beta(t), \quad (26)$$

$$w(t) = \beta(a(t)) \quad (27)$$

where $\beta(t)$ is determined from (24), $Y(t)$ from (3) and $L(t)$ from (4).

The proof is similar to the one provided in Boucekkine, Hritonenko, Yatsenko (2011) with some routine modifications. The new optimality condition (25) features the optimal R&D investment rule. It equalizes the actualized marginal cost of one unit of good invested in R&D, the right-hand side, and its marginal benefit, the left-hand side: Technological advances made at t benefit to all machines produced from this date during their lifetime, which explains the structure of (25). Unlike the benchmark, nothing obvious can be said about the implementability or not of the interior solution for given prices from $t=0$. We move to the decentralized equilibrium and its balanced growth paths in sake for more clarity.

Decentralized equilibrium

The equilibrium of this economy is described by the following system of equations in the unknowns $R, \beta, \mu, m, a, y, c, r$, and w

$$\frac{\dot{c}}{c} = r(t) - \rho \quad (D1)$$

$$\mu(t) = e^{-\int_0^t r(s) ds} \quad (D2)$$

$$y(t) = \int_{a(t)}^t \beta(\tau) m(\tau) d\tau, \quad (D3)$$

$$\int_t^{a^{-1}(t)} \mu(\tau) [\beta(t) - \beta(a(\tau))] d\tau = \mu(t) \beta(t), \quad (D4)$$

$$w(t) = \beta(a(t)) \quad (D5)$$

$$y(t) = c(t) + R(t) + \beta(t) m(t), \quad (D6)$$

$$L(t) = \int_{a(t)}^t m(\tau) d\tau = 1, \quad (D7)$$

$$bnR^{n-1}(t) \int_t^\infty \beta^{1-d}(\tau) m(\tau) \left[\int_\tau^{a^{-1}(\tau)} \mu(z) dz - \mu(\tau) \right] d\tau = \mu(t), \quad (D8)$$

$$\beta(\tau) = \left(d \int_0^\tau bR^n(v) dv + B^d \right)^{1/d}, \quad (D9)$$

under the given boundary conditions. Equality (D6) is the new clearing condition of the final good market. Equations (D8) and (D9) come from the endogeneity of technological progress, they are specific to the latter. Notice that just like in the benchmark model, given the initial investment profile, one can determine the scrapping time $a(t)$ from the labor market clearing condition (D7). But the similarity stops here: because technological progress is endogenous, the scrapping condition (D5) does not allow to compute wages. That's while the initial profile $R(t)$ does allow to compute the initial profile in $\beta(t)$, the computation of actual $\beta(t)$ requires a sequence of $R(t)$ values (by equation (D9)), which are determined by the forward-looking equation (D8). So we move to balanced growth paths.

Balanced growth paths: indeterminacy in levels

We will explore the possibility of exponential solutions for $R(t)$, while $m(t)$ and $t-a(t)$ are constant, to the system (D1)-(D9). First of all, we start with the following preliminary result: if $R(t)$ is exponential, then $\beta(t)$ is *almost exponential* and practically undistinguishable from an exponent at large t in the sense of the following lemma:

Lemma 2. (Boucekkine, Hritonenko, Yatsenko 2011). *If $R(t)=R_0e^{Ct}$ for some $\gamma>0$, then⁶*

$$\beta(t) \approx \bar{R}^{n/d} \left(\frac{bd}{\gamma n} \right)^{1/d} e^{\gamma t/d} \quad (28)$$

at large t . In particular, $\beta(t)=\bar{R}^{n/d} (bd / \gamma n)^{1/d} e^{\gamma t/d}$ if $bd \bar{R}^n = \gamma n B^d$.

Boucekkine et al (2011) actually studied a problem similar to the firm problem under consideration here, that is they postulated given prices. In particular, they showed that such a partial equilibrium problem cannot admit balanced growth paths in the cases $n>d$ and $n<d$.

⁶ For brevity, we will omit the expression “at large t ” when using the notation $f(t) \approx g(t)$

This property still holds in general equilibrium as one can guess. We therefore restrict our attention to the case $n = d$. The key result of this section is:

Proposition 5 *At $n = d$, the decentralized equilibrium (D1)-(D9) possesses an interior optimal regime (BGP)*

$$R(t) \approx \bar{R} e^{\gamma t}, \quad \beta(t) \approx \bar{R} (b / \gamma)^{1/d} e^{\gamma t}, \quad (\text{DG1})$$

$$t - a(t) = T = \text{const}, \quad m(t) = \bar{m} = 1/T, \quad (\text{DG2})$$

$$r(t) = \gamma + \rho = \text{const}, \quad w(t) = \bar{R} (b / \gamma)^{1/d} e^{\gamma(t-T)}, \quad (\text{DG3})$$

$$y(t) \approx \bar{R} \left(\frac{b}{\gamma} \right)^{1/d} \frac{1 - e^{-\gamma T}}{\gamma T} e^{\gamma t}, \quad (\text{DG4})$$

$$c(t) = \bar{R} \left\{ \frac{1}{T} \left(\frac{b}{\gamma} \right)^{1/d} \left[\frac{1 - e^{-\gamma T}}{\gamma} - 1 \right] - 1 \right\} e^{\gamma t}, \quad (\text{DG5})$$

where the constants γ and T are determined by the nonlinear equations (BG4) and

$$\gamma^{(1-d)/d} (\rho + \gamma d) e^{\gamma T} = d b^{1/d} \frac{1 - e^{-\rho T}}{\rho T}, \quad (\text{DG6})$$

which has a positive solution, at least, at small ρ and $b \leq 1$.

Namely, if $\rho \ll 1$ and $b \leq 1$, then:

- the rate γ is a unique positive solution $0 < \gamma < b$ of the nonlinear equation

$$\gamma^{1/d} + \rho \gamma^{(1-d)/d} / d = b^{1/d} e^{-\gamma T} + o(\rho), \quad (29)$$

- a positive T is uniquely determined from equation (BG4) by Lemma 1.
- the optimal consumption given by (DG5) is positive.

The proof is long and tricky, we report it in the appendix. The main result of the proposition is that BGPs still exist once technological progress is endogenized. The uniqueness properties are more subtle than in the benchmark and the first extension. Indeed, all the BGP variables are undetermined as it transpires from (DG4) and (DG5) for example. Strictly speaking, this is not surprising: The BGP systems are undetermined under endogenous technical progress since we add an unknown (the growth rate) to the same set of equations. As in traditional endogenous growth models (see Barro and Sala-i-Martin, 1995, chapter 4), all variables are undetermined in level: the BGP restrictions only determine ratios of variables. This is also the case here. Since T and the growth rate γ are uniquely determined, the ratios consumption to output, investment to output and R&D effort to output are also uniquely determined.

Beside indeterminacy, the endogenization of technological progress leads to the simultaneous and unique determination of capital lifetime and the rate of technological progress, which complicates dramatically the computations. Moreover, the BGP is proved to exist, and notably the scrapping time is finite along the BGP, only if the R&D productivity parameter b is bounded: $b \leq 1$. The condition $b \leq 1$ of Proposition 5 is sufficient but not necessary. Such a condition arises because of the way how the theorem is proved. As below numerical simulation shows, the BGP exists for larger values b as well. An interesting way to investigate why is to study the comparative statics of this BGP when b goes up. In particular, one would like to know how T , γ and the three ratios listed just above react to an increase in b . It is natural to expect that the endogenous technological progress rate γ is larger for larger values of the R&D efficiency b . It can be proved analytically for a reasonable range of model parameters.

Proposition 6 (comparative statics in b). *If $\rho \ll 1$ and a BGP exists, then the endogenous rate of technological progress γ increases as the R&D efficiency b increases.*

The proof. Let us rewrite equation (DG6) at $\rho \ll 1$ as

$$F(\gamma, b) = d\gamma^{1/d} + \rho\gamma^{1/d-1} - db^{1/d}e^{-\gamma T} = 0$$

By Proposition 1, γT increases in γ , so the last term of function F increases in γ . The first two terms also increase in γ , therefore, the function F increases in γ : $\partial F / \partial \gamma > 0$. Next, $\partial F / \partial b = -b^{1/d-1}e^{-\sqrt{2}\gamma} < 0$. By the Theorem about Implicit Function, $d\gamma/db = -\partial F / \partial \gamma / \partial F / \partial b > 0$, which proves that γ is larger for larger values b . The proposition is proved. **Q.E.D**

We cannot obtain such a simple monotonic result as Proposition 6 for the lifetime T because, by Proposition 1, T is not monotonic in γ . Namely, T decreases in γ for “small enough” γ , but increases for γ close to $1-\rho$ (and becomes infinite as $\gamma=1-\rho$). The question is when (and whether) the endogenous γ can grow close to $1-\rho$. One can notice that Proposition 6 gives no further details on the character of γ increase (in particular, whether γ increases indefinitely for indefinite b). Finding corresponding analytic conditions in the terms of given model parameters appears to be not possible. Applying various small parameter approximations to the nonlinear system (BG4) and (DG6) is inefficient because of the complex interplay of parameters b , ρ and d in the nonlinear equation (33).

So, to find more subtle comparative statics, we resort to numerical simulation. Namely, we have found an approximate solution of the system of two nonlinear equations (BG4) and (DG6) in γ and T for $d=0.5$ and multiple combinations of parameters ρ and b . The obtained values of γ and T are of obvious interest and help to identify the reasonable range of the important given parameter – the R&D efficiency b (which is difficult to estimate using statistics or any other means). The corresponding calculations are done in Matlab.

A numerical exploration of the covariations of technological progress and capital lifetime

To start, let $\rho = 10\%$ and the R&D efficiency b change from 0.02 to 10. The corresponding growth rate γ is shown in Figure 1a. It increases from 0 at $b = 0.01$ to $\approx 40\%$ and slows down later and saturates around the value $\gamma \approx 0.8$. At simulation provided, the rate γ never reaches the above mentioned “critical” value $1-\rho = 0.9$.

The corresponding capital lifetime T decreases fast at $b = 0.01$ to 9 years at $b = 0.1$ and later stabilizes around 3.6 years (see Figure 1b). As predicted theoretically by Proposition 1, the capital lifetime starts to increase at some large b (and corresponding to “large” γ) but the subsequent path is rather flat (from 3.5 years to 4.6 years when b changes 1.3 from to 10). A realistic range for b seems to be $[0.05, 0.2]$ as γ would then increase from $\approx 0.5\%$ to $\approx 10\%$ and T decreases from 27 years to 5.6 years. The simulation results are similar for $\rho = 5\%$ and shown on the same Figures 1a and 1b.

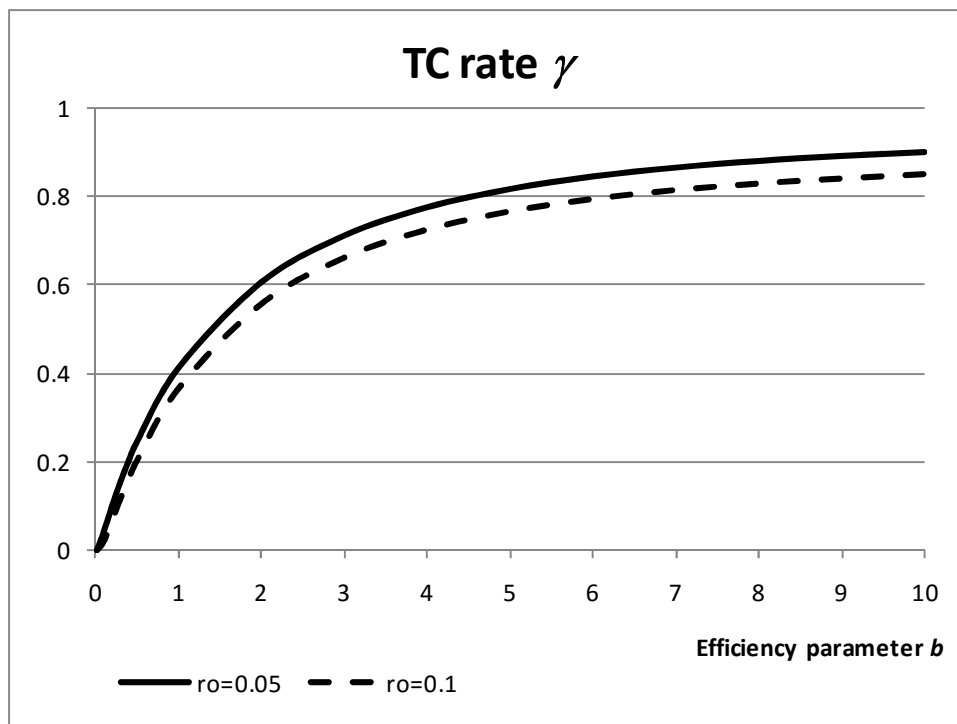


Figure 1a. Simulated TC rate γ for $d=0.5$ and b ranging from 0.01 to 10.

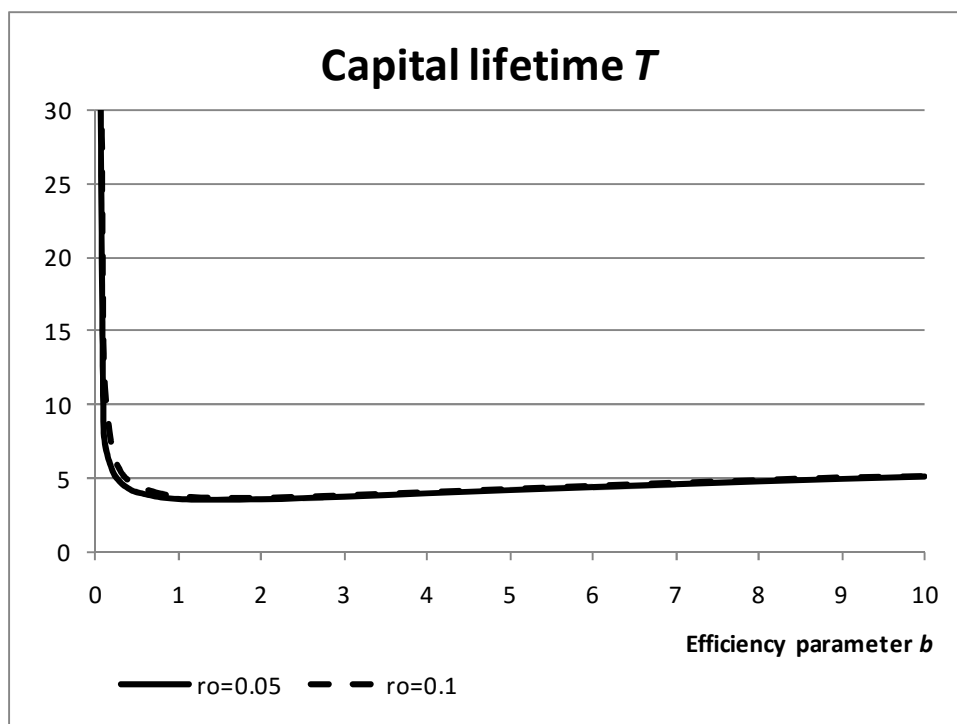


Figure 1b. Simulated optimal capital lifetime for $d=0.5$ and b ranging from 0.01 to 10.

5. Endogenous technical progress, employment and the lifetime of capital

We now consider the model incorporating the two extensions considered so far, that is putting together the consumer block with endogenous labor supply of Section 3, and the firm block with endogenous technical progress of Section 4. As in the previous section, we set $n=d$, and we seek for BGP solutions to the obtained dynamic system checking in particular $t-a(t)=T=\text{const}$ and $R(t)\approx\bar{R}e^{\gamma t}$. After the needed tedious computations, it turns out that the resulting BGP restrictions lead to

$$\beta(t) \approx \bar{R}(b/\gamma)^{1/d} e^{\gamma t}, \quad (\text{EG1})$$

$$t-a(t)=T=\text{const}, \quad m(t)=\bar{N}/T, \quad (\text{EG2})$$

$$r(t)=\gamma+\rho=\text{const}, \quad w(t)=\bar{R}(b/\gamma)^{1/d} e^{\gamma(t-T)}, \quad (\text{EG3})$$

$$\bar{y} = \bar{c} + \bar{R} + B\bar{m}, \quad (\text{EG4})$$

$$\Theta\bar{c} = \bar{R}\left(\frac{b}{\gamma}\right)^{1/d} e^{-\gamma T} \quad (\text{EG5})$$

$$y(t) \approx \bar{N}\bar{R}\left(\frac{b}{\gamma}\right)^{1/d} \frac{1-e^{-\gamma T}}{\gamma T} e^{\gamma t}. \quad (\text{EG6})$$

$$\gamma^{1/d} + \gamma^{(1-d)/d} \rho/d = \bar{N}b^{1/d} \frac{1-e^{-\rho T}}{\rho T} e^{-\gamma T}, \quad (\text{EG7})$$

where T is still given by equation (BG4). Needless to say, (EG7) is the counterpart of (DG6) when labor supply is endogenous. Substituting of m , y , and c from (EG2), (EG5), (EG6) into the good market equilibrium condition leads to the following equality:

$$\bar{R} \frac{\bar{N}}{T} \left(\frac{b}{\gamma}\right)^{1/d} \left[\frac{1-e^{-\gamma T}}{\gamma} - 1 \right] = \frac{\bar{R}}{\Theta} \left(\frac{b}{\gamma}\right)^{1/d} e^{-\gamma T} + \bar{R}, \quad (\text{EG8})$$

where the left-hand side is $\bar{y} - B\bar{m}$ (which is proportional to \bar{N}) and the right-hand side is $\bar{c} + \bar{R}$. After further transformations, we obtain the following system of three nonlinear equations with respect to the unknown \bar{N} , γ , and T , namely (BG4), (EG7) and equation (EG8) rewritten as:

$$\bar{N} \left[1 - \gamma - e^{-\gamma T} \right] = \frac{1}{\Theta} \gamma T e^{-\gamma T} + \gamma T \left(\frac{\gamma}{b} \right)^{1/d}. \quad (\text{EG9})$$

Compared to the previous extension in Section 2, we move from a set of two simultaneous equations determining \bar{N} and T , to a set of three simultaneous equations that determine \bar{N} , γ , and T . Since these equations are highly nonlinear, little can be proved analytically. This said, and before numerical experimentation, a few analytical things can be extracted from the system above. In particular, note that (EG9) gives:

$$\bar{N} = \frac{\gamma T}{1 - \gamma - e^{-\gamma T}} \left[\frac{e^{-\gamma T}}{\Theta} + \left(\frac{\gamma}{b} \right)^{1/d} \right].$$

Analytical comparison with the counterpart under exogenous technical progress is possible.

Proposition 7. *At the same rate of technical progress γ , employment is larger under endogenous technological progress.*

The proof is trivial. It's enough to compare the expression above with the corresponding formula under exogenous technological progress, namely equation (CG7):

$$\bar{N} = \frac{e^{-\gamma T}}{\Theta} \frac{\gamma T}{1 - \gamma - e^{-\gamma T}}.$$

This means that for the same value of the rate of technological progress, γ , inducing the same value for capital lifetime T by equation (BG4), the economy with endogenous technical progress is associated with more employment, which also implies, by $m(t) = \bar{N}/T$, that the former economy will experience a higher level of investment. In simple words, the presence of an innovative R&D sector in economy increases the willingness of people to invest and work at stationary equilibrium. It's easy to figure out why: in our model technological advancement is costly, the economy has to devote part of the final good to this activity, which (at the same intended rate of technological progress) means extra-labor supply compared to the case where innovations are exogenous since our technology is Leontief. Of course this nontrivial but partial result does not say much on the covariations of employment and technological progress in the extended economy, which we investigate numerically here below.

Numerical exploration of the covariations of technological progress and employment

In summary, a possible interior optimal regime (BGP) in the decentralized equilibrium model (EG1)-(EG7) is completely characterized by the system of three nonlinear equations (BG4), (EG7) and (EG8) with respect to $\bar{N} > 0$, $\gamma > 0$ and $T > 0$. If the BGP exists, then the optimal \bar{N} is always positive and may be interior, $0 < \bar{N} < 1$, or corner: $\bar{N} = 1$. In the case of full employment $\bar{N} = 1$, the BGP is equivalent to the problem of Section 4. However, the optimal \bar{N} will be interior $0 < \bar{N} < 1$, at least, at small b , which is confirmed by the numerical simulation below.

We assumed $\rho = 5\%$ at $d = 0.5$ and provided a series of simulations for the R&D efficiency b changing from 0.02 to 10 and the leisure parameter θ changing from 0.1 to 20. The results are shown in Figures 2 and 3. The figures demonstrate several interesting effects.

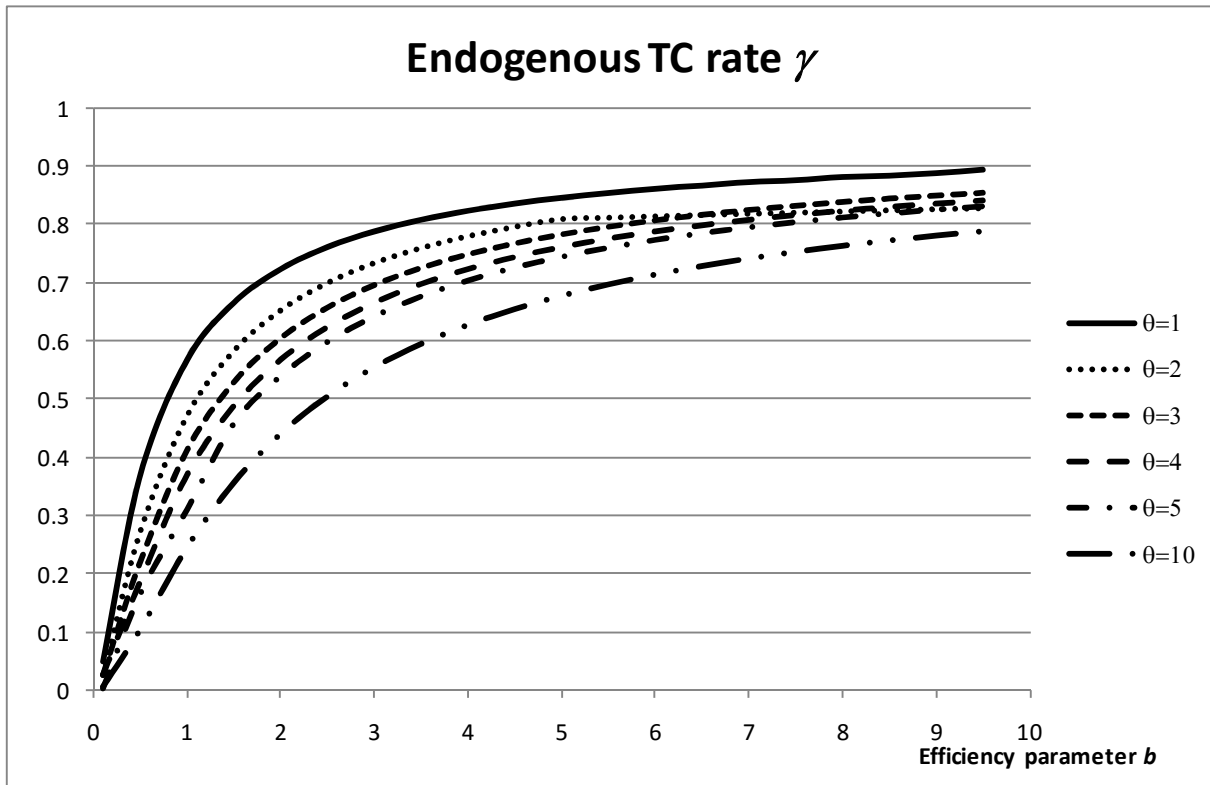


Figure 2a. The endogenous growth rate γ for $d = 0.5$, $\rho = 0.05$, and b ranging from 0.01 to 10.

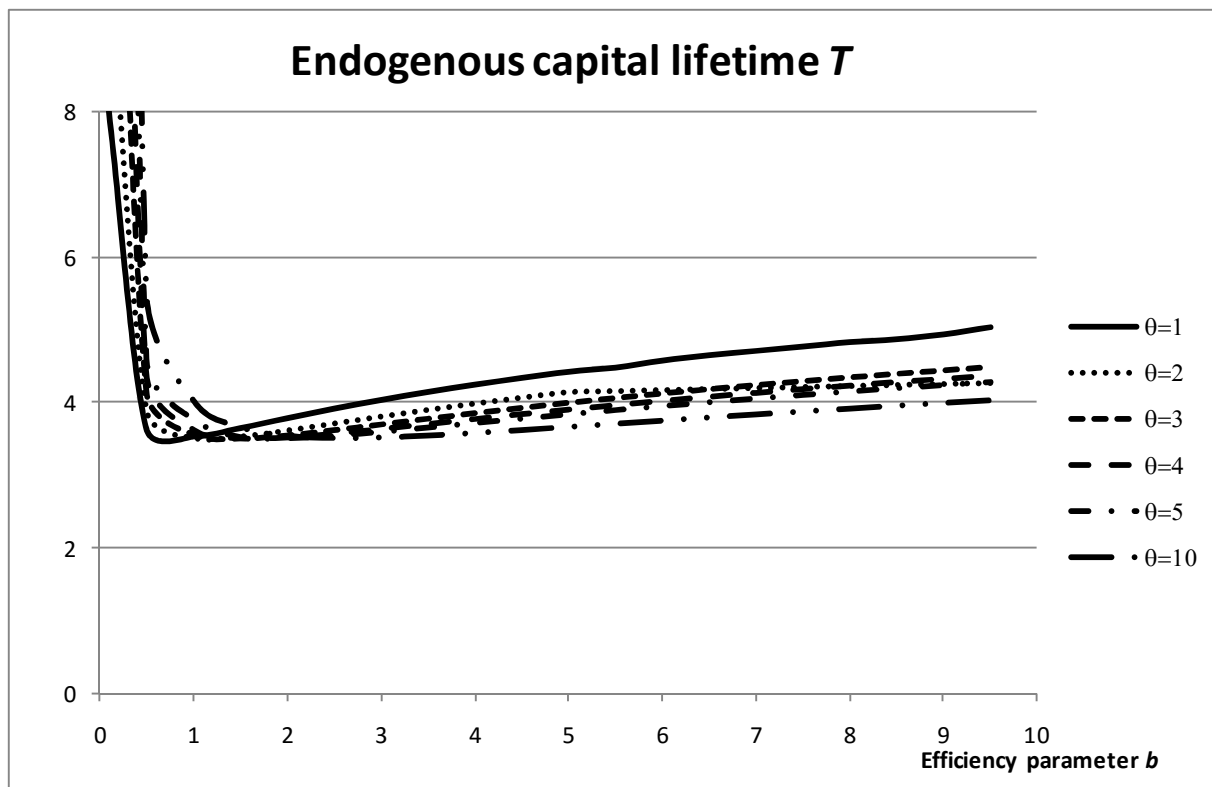


Figure 2b. The endogenous capital lifetime T for $d = 0.5$, $\rho = 0.05$, and $b = 0.01 \div 10$.

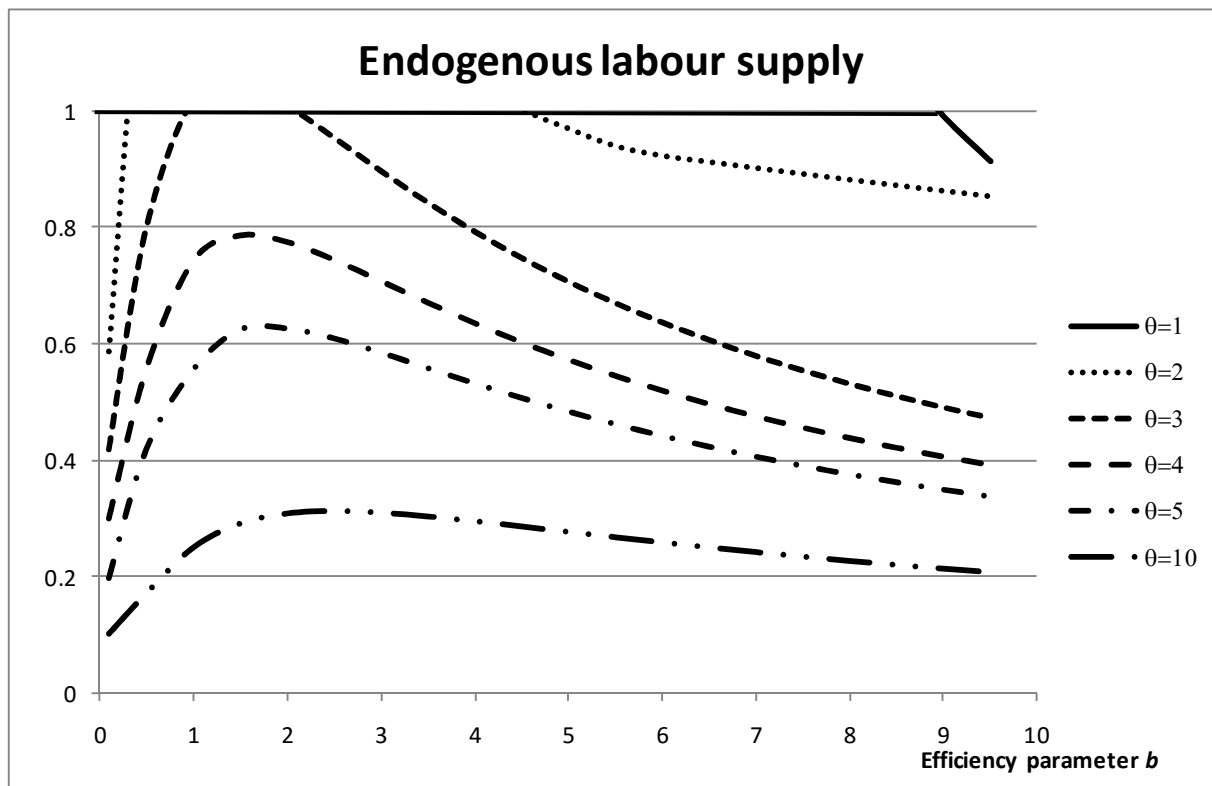


Figure 2c. Employment N for $d = 0.5$, $\rho = 0.05$, and $b = 0.01 \div 10$.

Figures 2a and 2b depict the behavior of the endogenous rate of technological progress γ and capital lifetime T when the R&D efficiency b increases. One can see that the basic qualitative picture is the same as in the model with fixed labor supply. Namely, Figures 2a and 2b resemble Figures 1a and 1b from Section 4 in the b ranges where that the optimal labor is interior: $\bar{N} < 1$. As in Figure 1b, the optimal capital lifetime T decreases fast at small b and starts to increase very slowly at larger b . However and in contrast to the previous models studied in Sections 3 and 4, both γ and T depend on θ now. In particular, as the preference of leisure increases, technological progress is shifted down. This is a natural outcome: increasing the rate of technological progress requires more final good, which itself requires more investment and more labor given labor/capital complementarity. As households are less willing to work, the resulting technological path goes down.

Figure 2c is new and nontrivial. First of all, it confirms the above theoretical findings that the optimal labor \bar{N} is the corner solution $\bar{N} = 1$ for not too large leisure parameter θ . Thus, the simulation shows that if $\theta \leq 0.2$, then the optimal labor is corner $\bar{N} = 1$ for any $0 < b < 22$. If $\theta = 0.5$, then $\bar{N} = 1$ for any $0 < b < 15$. If $\theta = 1$, then $\bar{N} = 1$ for $0 < b < 9$ and $\bar{N} < 1$ is interior for $9 \leq b$. Starting with $\theta \geq 4$, $\bar{N} < 1$ is always interior for any b , i.e., the labor is smaller when the value of leisure is high. Next, the dependence of the optimal labor \bar{N} on the R&D efficiency parameter b appears to be non-monotonic with a maximum reached at some value of b . Note that the usual calibration of the RBC models using indivisible labor typically yields $\bar{N} = 1/3$ (people work one third on their time). Non-monotonicity of the employment path is indeed apparent at all values of θ . Nonetheless, it is fair to observe that the turning points (that's the b values at which employment is maximal) correspond to γ values (in Figure 2a) which are close to 40% for reasonable values of \bar{N} . It follows that compensation does not always work in the vintage capital model: process innovation does create employment provided the innovation is not *too* radical.

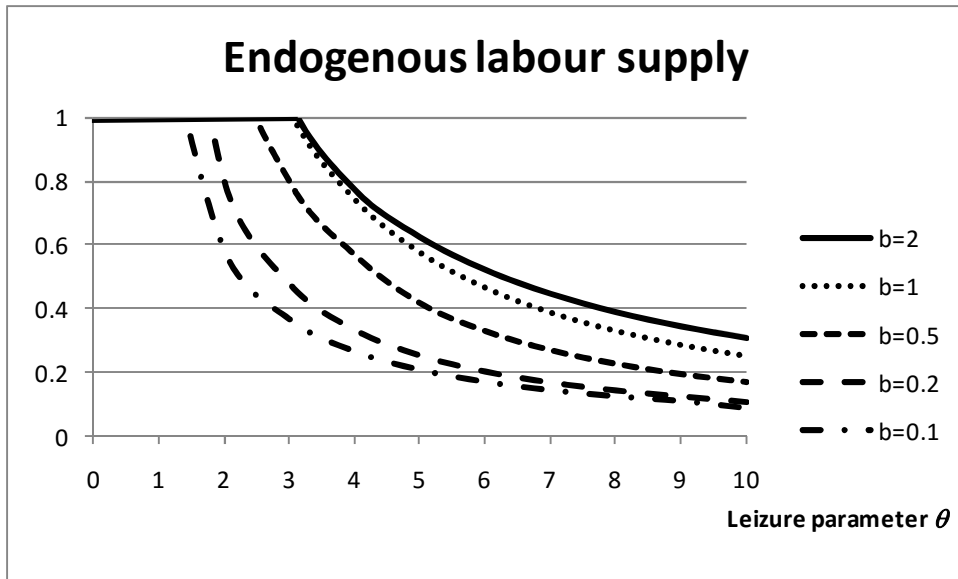


Figure 3a. Employment N for $d = 0.5$, $r = 0.5$ and θ ranging from 0.01 to 10.

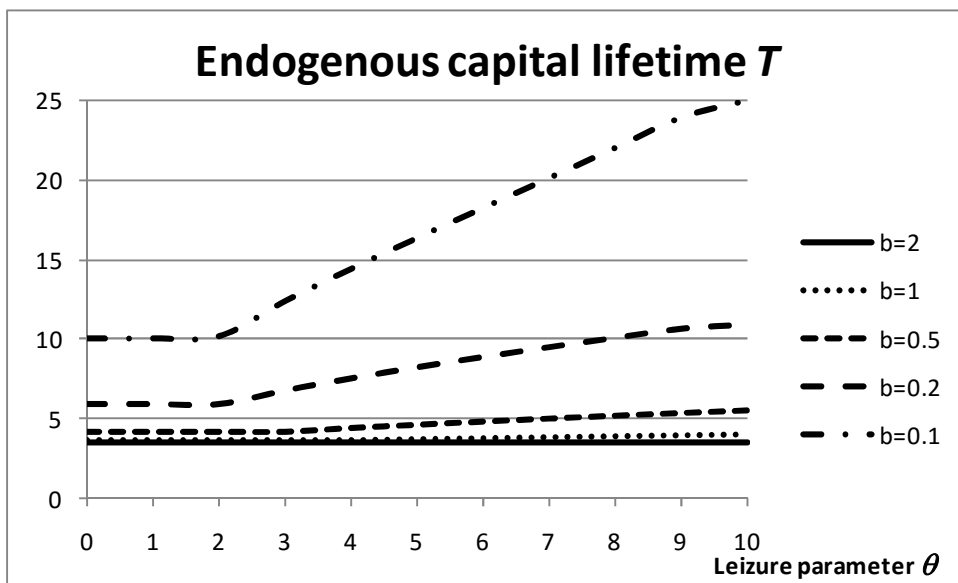


Figure 3b. The endogenous capita lifetime T for $d = 0.5$, $r = 0.5$ and $\theta = 0.01 \div 10$.

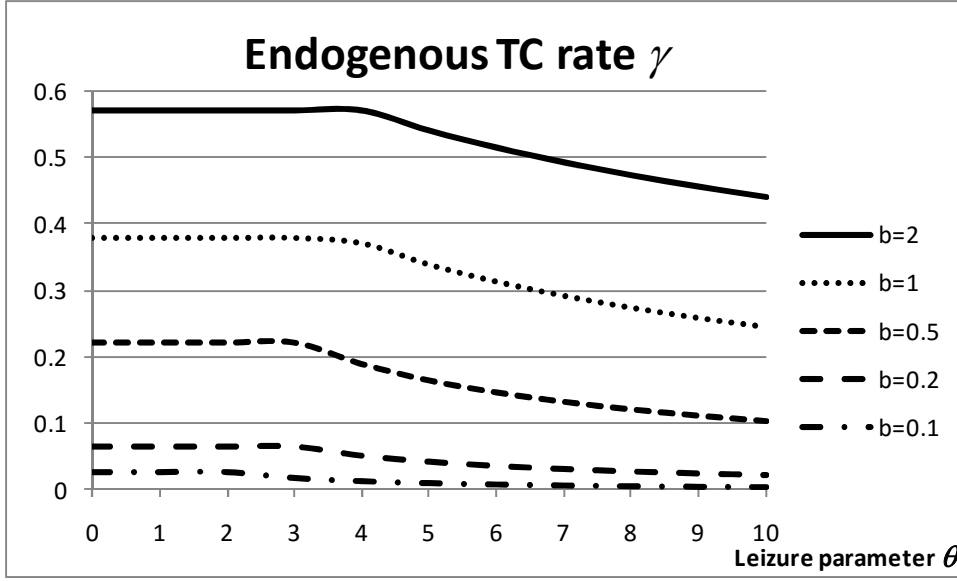


Figure 3c. The endogenous growth rate γ for $d = 0.5$, $r = 0.5$, and $\theta = 0.01 \div 10$.

Figures 3a-3c represent a different slice of the optimal picture when the leisure parameter θ increases (for different values of the R&D efficiency b). Figure 3a confirms that, for every given b , the optimal labor \bar{N} is corner $\bar{N}=1$ for the leisure parameter θ being less than some critical value θ_{cr} . The optimal labor \bar{N} is interior $\bar{N}<1$ and decreases if $\theta > \theta_{cr}$ and θ increases. The critical value θ_{cr} is larger for a larger b .

Figures 3b shows that the optimal capital lifetime T increases when θ increases. As households are less willing to work, equilibrium employment goes down, and given capital/labor complementarity, investment too, older machines have to be operated for a longer time. The increase is steeper for smaller values of b , and becomes negligible when b is large (thus, the increase is from 3.52 years to 3.55 years for $b=0.1$). Consistently, Figure 3c depicts the endogenous rate γ and is straightforward: γ decreases when the leisure parameter θ increases for the very same reasons.

6. Conclusion

In this paper, we have studied the impact of technological progress on the level of employment in a vintage capital model where: i) capital and labor are gross complementary; ii) labor supply is elastic; iii) there is full employment, and iv) the rate of labor-saving technological progress is endogenous. We restrict our analysis to the indivisible labor case. To this end, we have characterized the stationary distributions of vintage capital goods and the

corresponding equilibrium values for employment and the lifetime of capital. We have shown through successive extensions of a benchmark vintage capital model that both employment and the lifetime of capital are indeed non-monotonic functions of the rate of technological progress (when it is exogenous) or of the efficiency in the R&D sector (in the case of endogenous technological progress). In particular, we show that employment keep growing with the rate of technological progress unless innovations are too radical.

Studying the same problem for a general utility function and/or departing from the Leontief technologies (while assuring finite time scrapping for capital goods) is of course much more demanding. Despite our simplifying assumptions, we believe that our analysis makes already clear that the traditional “compensation problem” is definitely much less trivial when taking into account the vintage structure of capital, even though labor markets are taken efficient.

7. Appendix

Proof of Proposition 4: By Lemma 3, $\beta(t)$ is given by (28) at $n=d$. Then, the substitution of this relationship into equation (D8) at the BGP leads to

$$bd(\bar{R}e^n)^{d-1} \int_t^\infty \left(\bar{R} \left(\frac{b}{\gamma} \right)^{\frac{1}{d}} e^{\gamma\tau} \right)^{1-d} \frac{1}{T} \left[\frac{e^{-r\tau} - e^{-r(\tau+T)}}{r} - e^{-r\tau} \right] d\tau = e^{-rt},$$

and, after integration, to

$$\frac{db^{\frac{1}{d}} \gamma^{\frac{d-1}{d}}}{T[\gamma(1-d)-r]} \left[\frac{1-e^{-rT}}{r} - 1 \right] e^{-rt} = e^{-rt} \quad (A1)$$

Substituting $r = \gamma + \rho$ into (A1) and using equation (BG4) leads to (DG6).

The system of equations (BG4) and (DG6) may have a positive solution γ, T at natural assumptions. Namely, let $\rho < 1$. Then, presenting the exponent $e^{-\rho T}$ in (DG6) as the Taylor series, we obtain

$$\gamma^{(1-d)/d} (\rho + \gamma d) e^{\gamma T} = db^{1/d} + o(\rho) \quad (A2)$$

The function $F(\gamma) = \gamma^{(1-d)/d} (\rho + \gamma d) e^{\gamma T}$ monotonically increases to ∞ in γ and $F(0)=0$. Therefore, equation (A2) has a unique positive solution γ . By (A2), $(\gamma/b)^{1/d} (1 + \rho/\gamma d) e^{\gamma T} \approx 1$, therefore, $\gamma < b(1 + \rho/\gamma d)^{-d} \approx b(1 - \rho/\gamma)$. Since $b \leq 1$ in the theorem statement, then $\gamma < 1 - \rho$

and, by Lemma 1, equation (BG4) has a unique positive solution T at the known γ . At $\rho \ll 1$, presenting two exponents $e^{-\rho T}$ in (BG4) as the Taylor series, we have

$$1 - e^{-\gamma T} - \gamma T e^{-\gamma T} = \rho + \gamma + o(\rho) \quad (\text{A3})$$

The left-hand side of (A3) monotonically increases in γT and equals zero at $\gamma T = 0$ and $\gamma = 0$. Therefore, the equality (A3) establishes a one-to-one relationship between γT and γ .

If also $b \ll 1$, then the (A3) solution $\gamma \ll 1$ and $T \approx \sqrt{2/\gamma}$ by Lemma 1.

Knowing $\beta(t)$ and T , we can easily find the rest of the unknowns. In particular, let us look at $c(t)$ and analyze when $c(t) > 0$. We have

$$c(t) = y(t) - \beta(t)m(t) - R(t) \\ \approx \left\{ \bar{R} \left(\frac{b}{\gamma} \right)^{1/d} \left[\frac{1 - e^{-\gamma T}}{\gamma T} \right] - \bar{R} \left(\frac{b}{\gamma} \right)^{1/d} \frac{1}{T} - \bar{R} \right\} e^{\gamma t},$$

which gives (DG5). Now let us rewrite the latter as

$$c(t) = \bar{R} \left\{ \frac{1}{\gamma T} \left(\frac{b}{\gamma} \right)^{1/d} (1 - e^{-\gamma T} - \gamma) - 1 \right\} e^{\gamma t}, \quad (\text{A4})$$

Substituting the expression of γ through γT from the formula (A3) into (A4), we obtain

$$c(t) \approx \bar{R} \left\{ \left(\frac{\rho}{\gamma T} + e^{-\gamma T} \right) \left(\frac{b}{\gamma} \right)^{1/d} - 1 \right\} e^{\gamma t}.$$

Therefore, $c(t) > 0$ if

$$\left(\frac{\rho}{\gamma T} + e^{-\gamma T} \right) \left(\frac{b}{\gamma} \right)^{1/d} - 1 > 0. \quad (\text{A5})$$

By (29), $\frac{\rho}{\gamma d} + 1 = \left(\frac{b}{\gamma} \right)^{1/d} e^{-\gamma T}$ at $\rho \ll 1$. So, (A5) becomes

$$\frac{\rho}{\gamma d} + 1 + \left(\frac{b}{\gamma} \right)^{1/d} \frac{\rho}{\gamma T} > 0, \quad (\text{A6})$$

which holds for any positive ρ , γ , d , and T . The theorem is proved.

Q.E.D

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